NAG Toolbox for MATLAB

c05ad

1 Purpose

c05ad locates a zero of a continuous function in a given interval by a combination of the methods of linear interpolation, extrapolation and bisection.

2 Syntax

```
[x, ifail] = c05ad(a, b, eps, eta, f)
```

3 Description

c05ad attempts to obtain an approximation to a simple zero of the function $f(x)$ given an initial interval $[a,b]$ such that $f(a) \times f(b) \leq 0$. The same core algorithm is used by c05az whose specification should be consulted for details of the method used.

The approximation x to the zero α is determined so that at least one of the following criteria is satisfied:

(i) α lies within an interval $[\beta,\gamma]$ whose length satisfies $|\beta-\gamma| \leq 2 \times \textbf{eps} \times \max(|\beta|, 1)$,

(ii) $|f(x)| \leq \textbf{eta}$.

4 References

Brent R P 1973 *Algorithms for Minimization Without Derivatives* Prentice–Hall

5 Parameters

5.1 Compulsory Input Parameters

1: **a – double scalar**

a, the lower bound of the interval.

2: **b – double scalar**

b, the upper bound of the interval.

Constraint: $\textbf{b} \neq \textbf{a}$.

3: **eps – double scalar**

The absolute tolerance to which the zero is required (see Section 3).

Constraint: $\textbf{eps} > 0.0$.

4: **eta – double scalar**

A value such that if $|f(x)| \leq \textbf{eta}$, x is accepted as the zero. **eta** may be specified as 0.0 (see Section 7).

5: **f – string containing name of m-file**

f must evaluate the function f whose zero is to be determined.

Its specification is:

```
[result] = f(xx)
```

Input Parameters

1: **xx – double scalar**

The point at which the function must be evaluated.

Output Parameters

1: **result – double scalar**

The result of the function.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **x – double scalar**

The approximation to the zero.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **eps** ≤ 0.0,
or **a** $=$ **b**,
or $\mathbf{f}(\mathbf{a}) \times \mathbf{f}(\mathbf{b}) > 0.0$.

ifail $= 2$

Too much accuracy has been requested in the computation; that is, the interval containing the zero has been reduced to one of relative length at most ϵ, the ***machine precision***, but the exit conditions described in Section 3 are not satisfied. It is unsafe to continue reducing the interval beyond this point, but the final value of **x** returned is an accurate approximation to the zero.

ifail $= 3$

A change in sign of $f(x)$ has been determined as occurring near the point defined by the final value of **x**. However, there is some evidence that this sign-change corresponds to a pole of $f(x)$.

7 Accuracy

The levels of accuracy depend on the values of **eps** and **eta**. If full machine accuracy is required, they may be set very small, resulting in an exit with **ifail** $= 2$, although this may involve many more iterations than a lesser accuracy. You are recommended to set **eta** $= 0.0$ and to use **eps** to control the accuracy, unless you have considerable knowledge of the size of $f(x)$ for values of x near the zero.

8 Further Comments

The time taken by c05ad depends primarily on the time spent evaluating the function f (see Section 5).

If it is important to determine an interval of relative length less than **eps** containing the zero, or if the user-supplied real function **f** is expensive to evaluate and the number of calls to **f** is to be restricted, then use of c05az is recommended. Use of c05az is also recommended when the structure of the problem to be solved does not permit a simple function f to be written: the reverse communication facilities of c05az are more flexible than the direct communication of **f** required by c05ad.

9 Example

```
c05ad_f.m

function [f] = c05adf_f(x)
  f=exp(-x)-x;
```

```
a = 0;
b = 1;
eps = 1e-05;
eta = 0;
[x, ifail] = c05ad(a, b, eps, eta, 'c05ad_f')
```

```
x =
    0.5671
ifail =
           0
```